



# COMP 1023 Introduction to Python Programming

## Tutorial 3: Operations

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# Representing Numbers in Computers

- Most programming languages have at least **two** categories of data types to represent numbers
- **Integral** data types, which represent **integers** only
- **Non-integral** data types, which represent **decimals** (includes integers)
- Computer Organization (COMP 2611 or ELEC 2350) will go into more details how numbers are represented in memory

# Representing Integers in Computers

- **Integral** data types, which represent **integers** only
- In Python...
  - **int** takes a **dynamic** number of bytes
  - **int** can represent **arbitrarily small or large** integers
- In many other programming languages...
  - Often have a **fixed** number of bytes
  - Often can only represent a **finite range** of integers

# Representing Decimals in Python

- **Non-integral** data types, which represent **decimals** (includes integers)
- In Python...
  - **float** usually takes 8 bytes (64 bits)
  - Its representation, **IEEE 754**, is similar to non-integral data types in other programming languages
- There are **(uncountably) infinite** real numbers, so it is **impossible** for float to represent **every real number** using 8 bytes only
- The **nearest representable** real number is used
- **Precision loss** in **representation** and **calculations** when using float!
  - Python also has a rarely used data type **decimal.Decimal**, which does not suffer from precision loss up to a user-specified precision

## iPRS Question 1

What does the following code print?

```
value = float("3e14")  
print("Type:", type(value), sep="\n")
```

- A. Type:  
    <class 'float'>
- B. Type: 3000000000000000.0
- C. ValueError: could not convert string to float: '3e14'
- D. Type: <class 'float'>
- E. Type: <class 'int'>

## iPRS Question 1 - Answer

What does the following code print?

```
value = float("3e14")  
print("Type:", type(value), sep="\n")
```

- A. Type:  
    <class 'float'>
- B. Type: 3000000000000000.0
- C. ValueError: could not convert string to float: '3e14'
- D. Type: <class 'float'>
- E. Type: <class 'int'>

### Explanation

- 3e14 represents a number in **scientific notation**; scientific notation always use **float**
- The separator (sep) between each object provided to print is changed to **a newline** instead of a space

# Representing Decimals in Python

A classical example...

```
import math
```

```
print(1 + 2) # 3
```

```
print(1 + 2 == 3) # True
```

```
print(0.1 + 0.2) # 0.30000000000000004
```

```
print(0.1 + 0.2 == 0.3) # False
```

```
print(math.isclose(0.1 + 0.2, 0.3)) # True
```

```
print(abs((0.1 + 0.2) - 0.3) < 1e-10) # Similar to `math.isclose`  
# In general, use `math.isclose` to check for `float` equality.
```

Always keep this in mind when using **floats**!

Further reading: [Floating-Point Arithmetic: Issues and Limitations](#)

## iPRS Question 2

What does the following code print?

```
import math
x = math.pi      #  $\pi$ 
y = math.tan(x)   #  $\tan(\pi)$ 
print(y)          # ???
```

- A. Zero
- B. A nonzero value very far away from zero
- C. A nonzero value very close to zero

## iPRS Question 2 - Answer

What does the following code print? In mathematics,  $\tan(\pi) = 0$ .

```
import math
x = math.pi      #  $\pi$ 
y = math.tan(x)   #  $\tan(\pi)$ 
print(y)          # ???
```

- A. Zero
- B. A nonzero value very far away from zero
- C. A nonzero value very close to zero

### Explanation

$\pi$  requires an **infinite** number of digits to represent **exactly**, so it cannot be represented **exactly** by a float! A float in Python can represent 15 to 17 **significant digits**.

# Arithmetic Operators

- Refer to lecture slides
  - Arithmetic operations
- Addition: +
- Subtraction: -
- Multiplication: \*
- Division: /
- Floor division: //
- Exponentiation: \*\*
- Remainder: %

## Exercise: Arithmetic Operators

Convert the following natural language sentences into a one-line Python codes.

1. Add  $x$  to  $y$
2. Subtract  $x$  from  $y$
3. Multiply  $x$  by  $y$
4. (Demo) Divide  $x$  by  $y$
5. (Demo)  $x$  divides  $y$
6. Remainder of dividing  $x$  by  $y$
7. Square of  $x$
8. Square root of  $x$
9.  $x$ -th power of  $y$
10. The largest integer that is not larger than the result of dividing  $x$  by  $y$
11. (Challenge) The smallest integer that is not smaller than the result of dividing  $x$  by  $y$

## Exercise: Arithmetic Operators

Convert the following natural language sentences into a one-line Python codes.

1. Add x to y:  $x + y$  or  $y + x$  (addition is commutative)
2. Subtract x from y:  $y - x$
3. Multiply x by y:  $x * y$  or  $y * x$  (multiplication is commutative)
4. Divide x by y:  $x / y$
5. x divides y:  $y / x$
6. Remainder of dividing x by y:  $x \% y$
7. Square of x:  $x ** 2$  or  $x * x$
8. Square root of x:  $x ** 0.5$  or  $x ** (1 / 2)$
9. x-th power of y:  $y ** x$
10. The largest integer that is not larger than the result of dividing x by y:  $x // y$ 
  - This takes the **floor** of the result of division, i.e. **floor division**
11. (**Challenge**) The smallest integer that is not smaller than the result of dividing x by y:  
 $-(-x // y)$ 
  - This takes the **ceiling** of the result of division
  - Take a moment to see why the above code is correct. . .

## Exercise: Order and Associativity of Operations

Refer to lecture slides

- Precedence of arithmetic operators
- Associativity of arithmetic operators

Evaluate the following Python arithmetic expressions. No need to care about the return type for now.

1.  $1 + 2 * 3$
2. (Demo)  $(1 + 2) * 3$
3.  $1 + 2 + 3$
4.  $1 + (2 + 3)$
5.  $2 ** 2 ** 3$
6.  $(2 ** 2) ** 3$
7. (Demo)  $-1 ** 2$

## Exercise: Order and Associativity of Operations

Evaluate the following Python arithmetic expressions. No need to care about the return type for now.

1. `1 + 2 * 3`: 7

2. `(1 + 2) * 3`: 9

3. `1 + 2 + 3`: 6

4. `1 + (2 + 3)`: 6

5. `2 ** 2 ** 3`: 256

6. `(2 ** 2) ** 3`: 64

7. `-1 ** 2`: -1

- Exponentiation has a higher operator precedence than negation
- Consider writing the same expression in mathematics:  $-1^2 = -(1^2)$

# Return Type of Arithmetic Operators

- Only consider two possible **input** and **output** types: **int**, **float**
- **Not** consider operations that may return **other types**, e.g. **complex**
- **Rules of thumb** to help **memorize** return type: see **appendix**

## Exercise: Return Type of Arithmetic Operators

State the return type of the following expressions.

1.  $1 + 2$

2.  $7 / 4$

3.  $7 // 4$

4.  $7. // 4$

5.  $(1 + 2) * 3 * 4 * 1$

6. (Demo)  $(1 + 2) * 3 * 4 / 1$

7.  $(1 + 2) * 3 * 4 * 1e0$

8.  $4 ** 95$

9. (Demo)  $4 ** -95$

## Exercise: Return Type of Arithmetic Operators

State the return type of the following expressions.

1.  $1 + 2$ : `int`
2.  $7 / 4$ : `float`
3.  $7 // 4$ : `int`
4.  $7. // 4$ : `float`
5.  $(1 + 2) * 3 * 4 * 1$ : `int`
6.  $(1 + 2) * 3 * 4 / 1$ : `float`
  - The final operation is division ( $/$ ), which produces a float
7.  $(1 + 2) * 3 * 4 * 1e0$ : `float`
  - Numbers in `scientific notation` are always floats
8.  $4 ** 95$ : `int`
9.  $4 ** -95$ : `float`
  - Taking the `negative` power requires `division`

## iPRS Question 3

Which of the following inputs will make the following program produce a runtime error?

```
xx = float(input())  
print("Reciprocal is", xx ** -1)
```

- A. 4.2
- B. 1
- C. 0
- D. -1

## iPRS Question 3 - Answer

Which of the following inputs will make the following program produce a runtime error?

```
xx = float(input())  
print("Reciprocal is", xx ** -1)
```

- A. 4.2
- B. 1
- C. 0
- D. -1

### Explanation

$$0^{-1} = 1/0 = ???$$

# Assignment

- Refer to lecture slides
  - **Assignment**:  $\langle \text{var} \rangle = \langle \text{expr} \rangle$
  - **Simultaneous assignment**:  $\langle \text{var1} \rangle, \langle \text{var2} \rangle, \dots, \langle \text{varN} \rangle = \langle \text{expr1} \rangle, \langle \text{expr2} \rangle, \dots, \langle \text{exprN} \rangle$
  - **Cascading assignment**:  $\langle \text{var1} \rangle = \langle \text{var2} \rangle = \dots = \langle \text{varN} \rangle = \langle \text{expr} \rangle$
  - **Augmented assignment**:  $\langle \text{var} \rangle += \langle \text{expr} \rangle$  (similarly for other arithmetic operations)
- Simultaneous assignment and cascading assignment can be **used together**, e.g.  
 $\langle \text{var1} \rangle, \langle \text{var2} \rangle = \langle \text{var3} \rangle, \langle \text{var4} \rangle = \langle \text{expr1} \rangle, \langle \text{expr2} \rangle$
- Augmented assignment can only be used **by itself**

# Cascading Assignment

- $aa = bb = cc = 1 + 2$
- Equivalent to:  

```
temp = 1 + 2 # `1 + 2` is evaluated exactly once  
aa = temp  
bb = temp  
cc = temp
```
- The **rightmost hand side** expression is evaluated **once** and **saved**, and **before** any assignment to variables
- **Order** of assignment is from **left** to **right**
  - **Reverse** of that in many other programming languages, e.g. C++, Java, etc.

# Simultaneous Assignment

- A special case of **unpacking** (learn later)

- `aa, bb, cc = 1 + 2, 3, 4`

- Equivalent to:

```
temp0, temp1, temp2 = 1 + 2, 3, 4 # `1 + 2`, `3`, and `4` are each evaluated once
aa = temp0 # aa = 3
bb = temp1 # bb = 3
cc = temp2 # cc = 4
```

- The **rightmost hand side** expression is evaluated **once** and **saved**, and **before** any assignment to variables
- **Order** of assignment is from **left** to **right**

# Cascading Assignment and Simultaneous Assignment

- Cascading assignment and simultaneous assignment can be **used together**

- **Behaviors** of both assignments apply

- `aa, bb, cc = xx, yy, zz = 1 + 2, 3, 4`

- Equivalent to:

```
temp0, temp1, temp2 = 1 + 2, 3, 4 # `1 + 2`, `3`, and `4` are each evaluated once
```

```
aa = temp0 # aa = 3
```

```
bb = temp1 # bb = 3
```

```
cc = temp2 # cc = 4
```

```
xx = temp0 # xx = 3
```

```
yy = temp1 # yy = 3
```

```
zz = temp2 # zz = 4
```

- What about...

```
aa, bb, cc = uu = 1 + 2, 3, 4
```

- Revisit this later by yourself once you have learnt **collections**

# Augmented Assignment

- `aa *= 1 + 2`
- Equivalent to: `aa = aa * (1 + 2)`
- The variable must be **initialized**, as augmented assignment **reads** the variable first
- The **right hand side** expression is evaluated once **before** the assignment
  - **Not** equivalent to: `aa = aa * 1 + 2`
- Interesting fact: **No** increment (`++aa`, `aa++`) or decrement (`--aa`, `aa--`) operators in Python
  - In Python, resort to **augmented assignment**: `aa += 1`
  - Many other programming languages have them

## Exercise: Assignment

Give the values of all variables for each of the following codes. Consider each case separately.

1. `x = 42`

2. `x = x`

3. `x = 1023`  
`x = x`

4. `x = y = 1023`

5. `x, y = 10, 23`

6. (Demo)  
`x, y = 10, 23`  
`y, x = x, y`

7. (Demo)  
`x = 7`  
`x //= 3`

## Exercise: Assignment (Cont.)

Give the values of all variables for each of the following codes. Consider each case separately.

1. `x = 42`
2. `NameError: name 'x' is not defined`
3. `x = 1023`
4. `x = 1023; y = 1023`
5. `x = 10; y = 23`
6. `x = 23; y = 10`
7. `x = 2`

## Exercise: Assignment

Give the values of all variables for each of the following codes. Consider each case separately.

6. `x = 23; y = 10`

- Idiomatic (or as they say, 'Pythonic') way to `swap` two (or more) variables
- Equivalent to:

```
# x, y = 10, 23
temp0, temp1 = 10, 23
x = temp0 # x = 10
y = temp1 # y = 23
```

```
# y, x = x, y
temp2, temp3 = x, y
y = temp2 # y = 10
x = temp3 # x = 23
```

## iPRS Question 4

Which of the following assignment expressions produce an error?

- A. `course = favorite_course = "COMP 1023"`
- B. `subject, code = "COMP", "1023"`
- C. `comp *= 1023`
- D. `course = subject, code = "COMP", "1023"`

## iPRS Question 4 - Answer

Which of the following assignment expressions produce an error?

- A. `course = favorite_course = "COMP 1023"`
- B. `subject, code = "COMP", "1023"`
- C. `comp *= 1023`
- D. `course = subject, code = "COMP", "1023"`

### Explanation

`comp` is uninitialized.

## Exercise: Comparison Operators

Refer to lecture slides

- Relational operations
- String comparisons

Evaluate the following expressions involving comparison.

1. `8 < 24`
2. `8 ==> 24`
3. `1 = 1`
4. `-0.0 != 0.0`
5. `"COMP 1021" == "COMP 1023"`
6. `"COMP 1021" < "COMP 1023"`
7. `"ZZa" < "aZZ"`
8. (Demo) `"ZZa" < "ZZ"`
9. (Demo) `"2" < "10"`
10. (Supplementary) `import math; math.nan == math.nan`

## Exercise: Comparison Operators

Evaluate the following expressions involving comparison.

1. `8 < 24`: `True`
2. `8 => 24`: `SyntaxError: cannot assign to literal`
3. `1 = 1`: `SyntaxError: cannot assign to literal here. Maybe you meant '==' instead of '='?`
4. `-0.0 != 0.0`: `False`
5. `"COMP 1021" == "COMP 1023"`: `False`
6. `"COMP 1021" < "COMP 1023"`: `True`
7. `"ZZa" < "aZZ"`: `True`
  - All **capital** letters are lexicographically smaller than all **lowercase** letters in **ASCII**
8. `"ZZa" < "ZZ"`: `False`
  - If a str is a prefix of the other str, the **shorter** str is lexicographically smaller
9. `"2" < "10"`: `False`
  - Since they are strs, we use **lexicographic order**
  - '2' comes after '1' lexicographically, so the first str is lexicographically larger

## Exercise: Comparison Operators

Evaluate the following expressions involving comparison.

10. (Supplementary) `import math; math.nan == math.nan`: **False**

- Surprise!
- `math.nan` stores a float representing NaN (not a number)
- NaNs are defined to be not equal to itself by IEEE 754
- Indeed, `math.nan != math.nan` is **True**
- All other comparison operators (except for `!=`) between two NaNs are also defined to return **False**

# Logical Operators

- Refer to lecture slides
  - Logical operations
- Left-associative
- Logical operator precedence
  1. not
  2. and
  3. or
- Logical expressions are more readable when consecutive operations using the same logical operator are parenthesized
  - Even in cases in which it is unnecessary by operator precedence
  - `a or b and c and d or e and f`
  - `a or (b and c and d) or (e and f)`
  - Equivalent, but their readability differs

## Exercise: Logical Operators

Evaluate the following expressions involving logical operators.

1. `0 == 9 or 1 == 1`
2. `1 < 2 and 2 < 3`
3. `not (not 1 < 2 or not 2 < 3)`
4. (Demo) `not not not not not 824 == 824`
5. (Demo) `True or True and False`
6. `1 < x or 1 >= x`, where `x` is an `int`

## Exercise: Logical Operators

Evaluate the following expressions involving logical operators.

1. `0 == 9 or 1 == 1`: **True**
2. `1 < 2 and 2 < 3`: **True**
3. `not (not 1 < 2 or not 2 < 3)`: **True**
  - `not` has a higher precedence than `and` and `or`
  - This item is equivalent to the previous item; left as an exercise to the reader
4. `not not not not not 824 == 824`: **False**
  - Count the number of `not`s
5. `True or True and False`: **True**
  - `and` has a higher precedence than `or`
  - Equivalent to `True or (True and False)`, but **not** equivalent to `(True or True) and False`
6. `1 < x or 1 >= x`, where `x` is an `int`: **True**
  - Mathematically, `x` is either smaller than 1 or not smaller than 1

# Lab 3

- This is not a hand-in lab: **No need** to submit anything
- Point-of-sale software
- Two paths: basic, advanced
- Basic: **recommended**; use a **console** to accept inputs and print outputs
- Advanced: **optional**; use a **graphical user interface** (GUI), like most of your applications
  - Uses **Tkinter**, a **GUI framework** that comes with Python

# Lab 3

- **Basic path:** recommended
- Task 1: Get product information from user input
  - How to get **user input** in a console?
  - How to **store** user input?
  - What is the original **type** of user input?
  - What should the **correct types** be according to the task requirements?
  - How to **convert** user inputs into the correct types?
- Task 2: Calculate the total cost of one kind of item
  - How many kinds of items are there?
  - Given the **unit cost** and **quantity** of an item, how to calculate its **total cost**?

# Lab 3

- **Basic path**: recommended
- Task 3: Calculate the total cost of both items
  - Given the total cost for **each item**, how to get the total cost of **all items**?
- Display the product information and total cost
  - What is the **output format** required by the task?
  - How to **format** a string? Note there are multiple ways.
  - What is the **best way** to format a string in this situation?

## Lab 3

- **Advanced path**: optional
- Same tasks as basic path
- However, each task is on separate **function** (learn later)
  - Helps to group closely related code together
  - Helps to reuse code
- Run **main\_gui.py** after finishing all tasks to open a GUI: **python main\_gui.py**
- Interact with the GUI to **check** your code
- A brief **Tkinter** tutorial on the lab webpage
  - A **GUI framework** that comes with Python
  - **Not needed** to finish the advanced path
  - For learning to create **your own GUIs** using Python

# Use of Numerical Data Types in Python

- `int`, `float`, `decimal.Decimal`
- When and where to use them?
- If your numerical data is integer, use `int`
  - No precision problems
  - Arbitrarily small or large integers can be represented
- Otherwise, use `float` or `decimal.Decimal`, depending on your use case
  - Most applications do not require precision
  - Thus `float` is commonly used
  - Still, apply some caution!

# Use of Numerical Data Types in Python

- A few **specialized** applications **require precision**
- **Banking**: If I add 0.1 dollars to my account, then 0.2 dollars, and finally remove 0.3 dollars, I expect 0 dollars left, not  $5.551115123125783e-17$  dollars.
  - Try running  $0.1 + 0.2 - 0.3$  in Python REPL
  - An example in later slides
- **Finance**: ... In January 1982 the index was initialized at 1000 ... Over the weekend of November 25–28, 1983, the error was corrected, raising the value of the index from its Friday closing figure of 524.811 to 1098.892. ([Vancouver Stock Exchange §Rounding errors on its Index price](#))
- **Mission-critical systems**: On 25 February 1991, a **loss of significance** in a MIM-104 Patriot missile battery prevented it from intercepting an incoming Scud missile. . . ([Floating-point arithmetic §Incidents](#))

## decimal.Decimal

- Import from the Python builtin package `decimal` first: `import decimal`, `from decimal import Decimal`, or `from decimal import *`
- Like `float`, it supports all `arithmetic` and `comparison` operators
- No `precision loss` up to a `user-specified precision` (number of accurate digits)
  - By default this is 28
  - Can be set using `decimal.getcontext().prec = desired_precision`
- Supports `conversion` from `int`, `float`, `str`, ...
  - Converting from a `float` uses its `exact` value, including any `precision loss`
    - `Decimal(3.14)` is `Decimal('3.140000000000000124344978758017532527446746826171875')`, because 3.14 cannot be represented `exactly` using `float`
  - Convert from a `str` instead to avoid this
    - `Decimal("3.14")` is `Decimal('3.14')`
- Explore by yourself!

## decimal.Decimal

- Revisit the banking example

- Bad version using `float`:

```
balance = 0.0
while True:
    print("You have", balance, "dollars in your bank account")
    change = float(input("Deposit (or withdraw) money: "))
    balance += change
```

- Better version using `decimal.Decimal`:

```
import decimal
balance = decimal.Decimal("0")
while True:
    print("You have", balance, "dollars in your bank account")
    change = decimal.Decimal(input("Deposit (or withdraw) money: "))
    balance += change
```

- For both versions: deposit 0.1 dollars, and then deposit 0.2 dollars
- Exit the program by pressing `Ctrl+C`

# Other Things

- Topics covered in lectures but not this tutorial
- Refer to lecture slides
  - Introduction
  - Operator precedence table
  - Division by zero
  - Chain operators: also see [appendix](#)
  - Membership operations
  - Floating point errors
  - Common pitfalls

That's all!

Any question?



# Appendix

- These materials are optional
- Feel free to read them if you are interested

# A Brief Etymology of 'Computer'

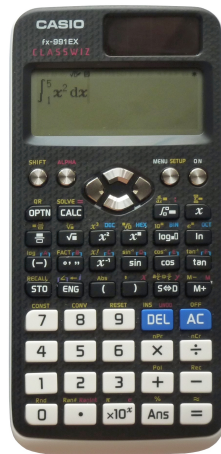
- 'Computer' used to refer to a person who carries out calculations
- It was not until the mid-20th century that 'computer' acquired its current definition
- Nowadays computers (its modern definition) can perform billions of calculations every second!
- You do not need to be a 'human computer', but you should definitely know how to instruct computers to perform arithmetic operations



NACA High Speed Flight  
Station 'Computer Room'

# A Physical Calculator

- A calculator may include:
  - Basic arithmetic: addition (+), subtraction (−), multiplication (×), division (÷), exponentiation, ...
  - Elementary functions: sine, cosine, tangent, their inverses, logarithm, ...
  - Calculus: differentiation, integration, ...
  - Statistics: max, min, mean, median, variance, linear regression, ...
  - Graphics: graph of a function, ...
  - Memory and programs
  - ... and more!
- Python (with the appropriate packages) can also perform them as well!
  - With **great power** comes with **great complexity**...
  - For simplicity, let's only consider **basic arithmetic** first!



# Return Type of Arithmetic Operators

- **Rules of thumb** to help **memorize** return type (lower number is more **important**)
  1. It should **reasonably** represent the exact result
  2. Return **int** if both numbers are **int**, otherwise **float**
- **Examples**
  - $3 / 2$ : 1.5 (**float**); division in general does not produce an integer value, so return **float**
  - $3 // 2$ : 1 (**int**); floor division in general produce an integer value, so return **int**
  - $3.0 // 2$ : 1.0 (**float**); one operand is **float**, so return **float** even if the result is an integer value
  - $1 ** -1$ : 1.0 (**float**); taking the negative power requires **division**

# Order of Operations

- Refer to lecture slides
  - Precedence of arithmetic operators
- Python follows the same convention as that we humans normally use:
- Mnemonics for order of operations
  - PEMDAS: Parentheses, Exponents, Multiplication/Division, Addition/Subtraction
    - Please Excuse My Dear Aunt Sally
  - BODMAS: Brackets, Of, Division/Multiplication, Addition/Subtraction
    - "Of" meaning fraction multiplication
  - 先乘除後加減: "first multiply and divide, then add and subtract"

# Short Circuiting of Logical Operators

- To disable **short circuiting**. . .
  1. Any non-**bool** expressions should be converted to **bool** first using **bool(<expr>)**
  2. Replace **and** with **&**
  3. Replace **or** with **|**
  4. Add parentheses to ensure the **order of operations** is the same as before, as **&** and **|** have **higher operator precedences**
- Examples
  - **x or y** becomes:  
`x | y`
  - **x + 1 == y and z + 1 == w** becomes:  
`(x + 1 == y) & (z + 1 == w)`
  - **For exposition only: a\_list and a\_tuple** becomes:  
`bool(a_list) & bool(a_tuple)`

# Chaining Comparison Operators

- Refer to lecture slides
  - Chain operators
- It allows you to write **intuitive** comparison statements such as  $2 < x < y < 4$ 
  - **True** if  $x$  is between 2 and  $y$  (excluding 2 and  $y$ ) and  $y$  is between  $x$  and 4 (excluding  $x$  and 4)
- It also allows you to write **nonsensical** comparison statements such as  $-2 < x < y < -4$ 
  - **Don't** do this in practice!
- Many other programming languages do **not** allow this, e.g. C++, Java, ...
- An **arbitrary** number of **arbitrary** comparison operators can be chained (but usually **not a good idea**)
  - $2 < x < 4$
  - $2 < x < y < 4$
  - $2 == x < y == 4$ 
    - No good reason to write code like this in practice apart from exams...
- For the general form: see **appendix**

# Chaining Comparison Operators

- General form
  - $\langle \text{expr1} \rangle \langle \text{op1} \rangle \langle \text{expr2} \rangle \langle \text{op2} \rangle \dots \langle \text{opN-1} \rangle \langle \text{exprN} \rangle \langle \text{opN} \rangle \langle \text{exprN+1} \rangle$
- Interpretation
  - Each  $\langle \text{opK} \rangle$  corresponds to a comparison
    - $\langle \text{expr1} \rangle \langle \text{op1} \rangle \langle \text{expr2} \rangle$
    - $\langle \text{expr2} \rangle \langle \text{op2} \rangle \langle \text{expr3} \rangle$
    - ...
    - $\langle \text{exprN} \rangle \langle \text{opN} \rangle \langle \text{exprN+1} \rangle$
  - All of them must be **True** for the entire logical expression to be **True**; otherwise it is **False**
    - You can think that each of the above comparisons are parenthesized, and then joined together using **and**
  - Each  $\langle \text{exprK} \rangle$  is evaluated **exactly once**
    - Ignore this for now if you do not understand; it won't matter yet

# Chaining Comparison Operators

- The conversion is **imperfect**: Each `<exprK>` should be evaluated **exactly once**, but this is not the case after the conversion; see **appendix**
- `2 < x < 4`: `(2 < x) and (x < 4)`
- `2 < x < y < 4`: `(2 < x) and (x < y) and (y < 4)`
- `2 == x < y == 4`: `(2 == x) and (x < y) and (y == 4)`
- `2 < x < y < 4 > 6 != 7 == 9 >= -1 != 2 > 2`:  
(  
    `(2 < x) and (x < y) and (y < 4)`  
    `and (4 > 6) and (6 != 7) and (7 == 9)`  
    `and (9 >= -1) and (-1 != 2) and (2 > 2)`  
)

## Exercise: Chaining Comparison Operators

Evaluate the following expressions involving chained comparison operators.

1.  $2 < 3 < 4$
2. `"COMP 1021" < "COMP 1023" < "COMP 2011" < "COMP 2012"`
3.  $2 < x > 2$ , where  $x$  is an `int`

## Exercise: Chaining Comparison Operators

Evaluate the following expressions involving chained comparison operators.

1.  $2 < 3 < 4$ : **True**
2. `"COMP 1021" < "COMP 1023" < "COMP 2011" < "COMP 2012"`: **True**
3.  $2 < x > 2$ , where  $x$  is an **int**: **False**
  - Mathematically,  $x$  cannot be simultaneously smaller than 2 and larger than 2

# Meaning of “Evaluated Once”

- In **assignment** and **chaining comparison operators**, “evaluated once” is mentioned
- What does it **mean**? Why does it **matter**?
- Consider the following program:

```
if 0 <= float(input("Enter a number: ")) <= 10:  
    print("Your number is in between 0 and 10!")  
else:  
    print("Your number is not in between 0 and 10.")
```

- `float(input("Enter a number: "))` is “evaluated once”
- If you run the above program, you are prompted for a number **once**
- Consider the following “**equivalent**” program:

```
if 0 <= float(input("Enter a number: ")) and float(input("Enter a number: ")) <= 10:  
    print("Your number is in between 0 and 10!")  
else:  
    print("Your number is not in between 0 and 10.")
```

- If you run the above program instead, you are prompted for a number **twice**

# Meaning of “Evaluated Once”

- The example relates to **chaining comparison operators**, but the concept applies to **assignment** too
- “Evaluated once” matters when the expression has **side effects**
  - The expression `1 + 2` has no **side effects**; running it once or multiple times does not affect the **program behavior to the user** (apart from **performance**)
  - The expression `input("Enter a number: ")` has **side effects**; running it once or multiple times prompts the user for a number of **a different number of times**
- **Most** of the expressions we have learnt so far do not have **side effects**
- This is why this is **supplementary content**, because it **likely** does not matter **for now**
- Later, you will learn more expressions with side effects

# Value Equality and Object Equality

- Computer memory stores many **objects**; each object has a **memory address**
  - A city has many **houses**; each house has an **address**
- If two variables refer to the **same memory address**, they must refer to the **same object**
- The identity operator **is** (**is not**) tells us if this is the case; it checks for **object equality**
- The equals operator **==** checks for **value equality**

# Value Equality and Object Equality

- Semantically, they are different (their meanings are different)
- An object should equal the object itself in value
  - Object equality implies value equality
  - Side note: Python is naughty. . . you can change the behavior of `==` to always return `False` or do other weird things
- Different objects can have the same value
  - Value equality may not imply object equality
  - Imagine two `int` objects at different memory address, but they have the same value of 1023

# Special Values and Cached Values

- Special values with **unique** object or memory address
  - Python **guarantees** only **one** object with this **value** can **ever** exist
  - Thus in this **special case**, value equality **does** imply object equality
  - Examples: **None** (NoneType), **Ellipsis** or **...** (EllipsisType), ...
- Cached values
  - Whenever appropriate, Python avoids creating **new** objects, and **reuse** existing ones with the **same value** as the object to be created to **save memory**
  - An **implementation detail** of Python; **not guaranteed**
  - **Don't** rely on it for program **correctness**
    - Python may print a **SyntaxWarning** when you rely on it
    - Try running **0 is 0** in Python REPL
  - Examples: small **ints**, short **strs**, ...